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UNPUBLISHED PRELIMINARY DATA

THE USE OF AN OSCILLOSCOPE FOR THE MEASUREMENT OF SNR

by

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[redacted]

1388-11 15 January 1964

Prepared for
National Aeronautics & Space Administration
Washington, D.C., 20325

Department of ELECTRICAL ENGINEERING

THE OHIO STATE UNIVERSITY
RESEARCH FOUNDATION
Columbus, Ohio

RC #1

REPORT
by
THE OHIO STATE UNIVERSITY RESEARCH FOUNDATION
COLUMBUS, OHIO 43212

Sponsor	National Aeronautics and Space Administration 1520 H Street, N.W. Washington, D.C. 20325
Grant Number	NsG-213-61
Investigation of	Theoretical and Experimental Analysis of the Electromagnetic Scattering and Radiative Properties of Terrain, with Emphasis on Lunar-Like Surfaces
Subject of Report	The Use of an Oscilloscope for the Measurement of SNR
Submitted by	Jon W. Eberle Antenna Laboratory Department of Electrical Engineering
Date	15 January 1964

ABSTRACT

A technique is described for obtaining the signal to noise ratio of a signal in the presence of noise. The only instrument required is an oscilloscope. The accuracy of the method is dependent upon the ability of the observer to determine the point of peak intensity of the displayed waveform.

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THE USE OF AN OSCILLOSCOPE FOR THE MEASUREMENT OF SNR

INTRODUCTION

One of the most elusive parameters to measure in the laboratory or field is that of signal-to-noise ratio (SNR). Perhaps the foremost reason for this difficulty is that one of the variables comprising the parameter is a random waveform, i.e., the noise. Because of this, a statistical approach must be considered, at some point in the measurement process, in which a mean or mean square value is sought. Since most commonly available laboratory instruments are designed for use with sinusoidal waveforms, different techniques or interpretations must be used when measuring noise with conventional instruments. A particular application has been encountered in which the SNR was desired in the IF strips of an antenna array system[1]. In addition to needing a very accurate measure of the SNR, a quick check was required to ascertain proper equipment operation during data collection. Since an oscilloscope is normally used to monitor operation at all times, a technique has been developed which will yield a rather accurate measure of the SNR using only the scope presentation of the output signals.

MEASUREMENT TECHNIQUE

It is widely known that the envelope of narrow-band Gaussian noise has an amplitude probability density function given by the Rayleigh distribution[2]

$$q(r) = \frac{r\, e^{-r^2/2N}}{N}, \tag{1}$$

where r is the value of the envelope and N is the mean square value of the Gaussian noise. This density function is displayed in Fig. 1 for a normalized mean square value. As contrasted with the density function of the envelope amplitude of the narrow-band noise, its amplitude density function is shown in Fig. 2, along with the amplitude density function of a sine wave and broadband noise, for comparison purposes[3]. As can be seen from the density function in Fig. 2(b),

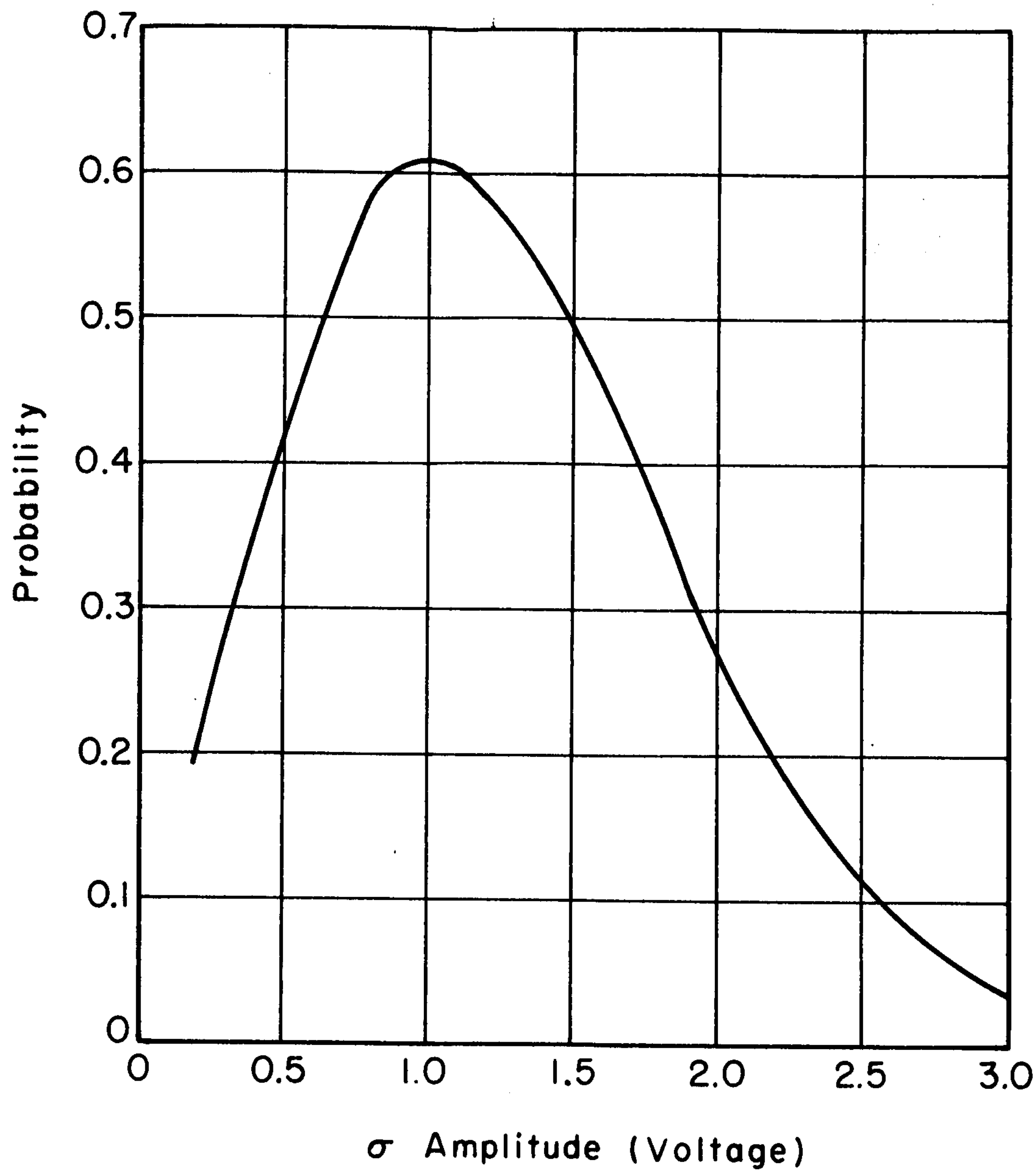


Fig. 1. Probability density function of envelope of narrow band Gaussian noise.

a portion of the energy around the $\sigma = 0$ value, where σ is the rms value of the random waveform, is distributed as is a sine wave of the same frequency as the mean frequency of the narrow-band noise. In other words, the narrow-band noise has a sinusoidal component. Utilizing this fact, an oscilloscope can be used to display the noise waveform when the scope is synchronized in an external or an internal triggering mode. For example, consider a narrow-band

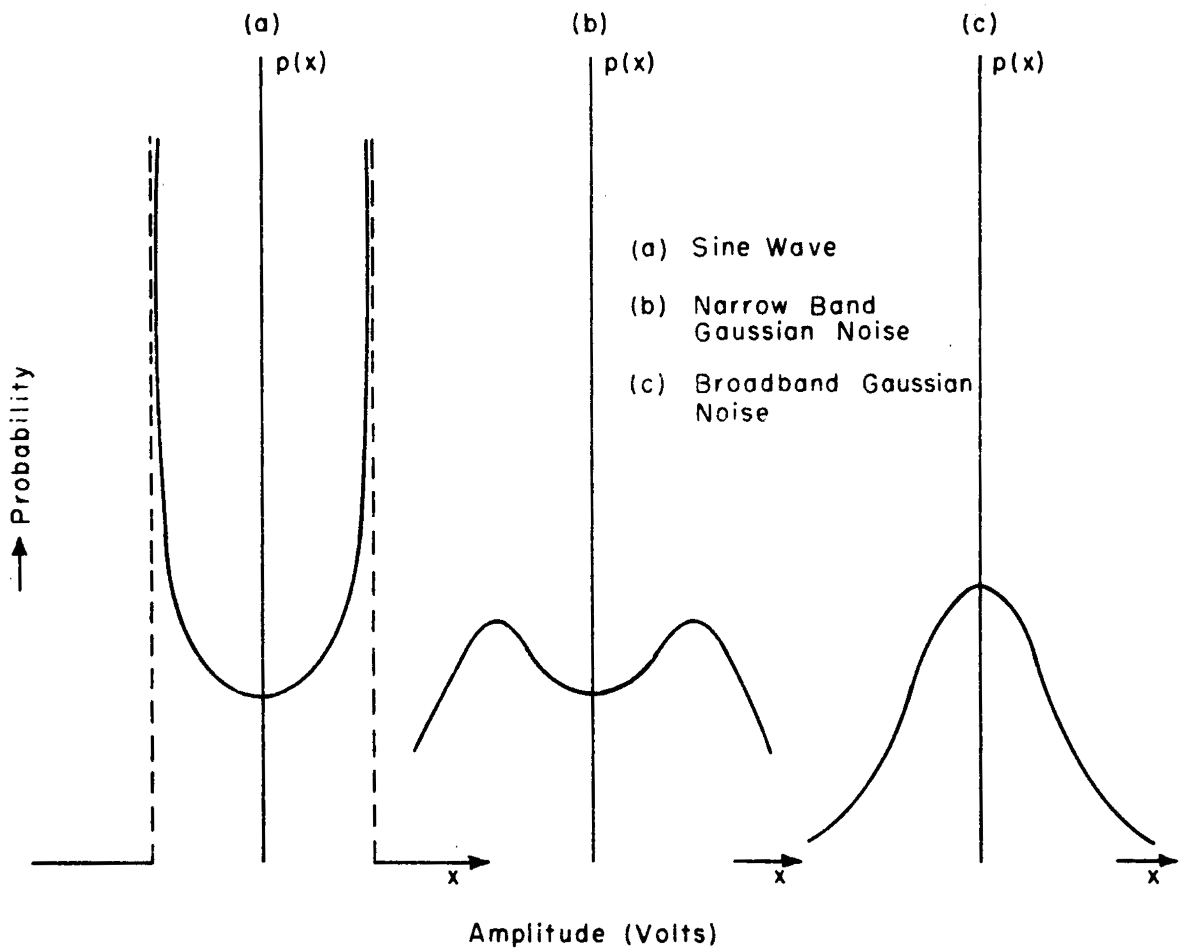
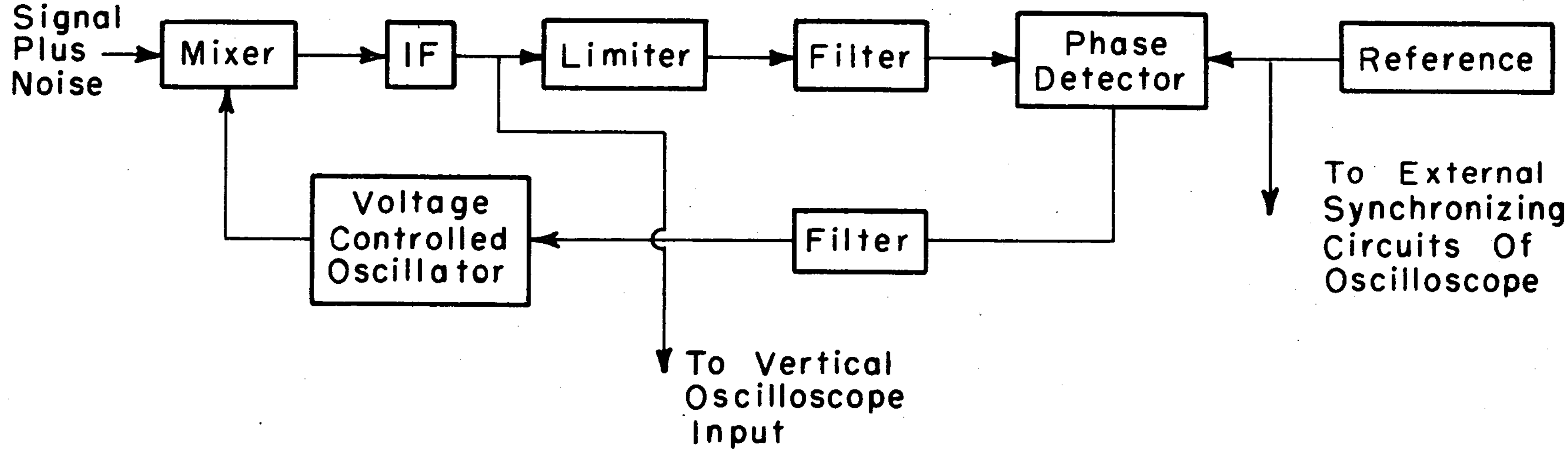
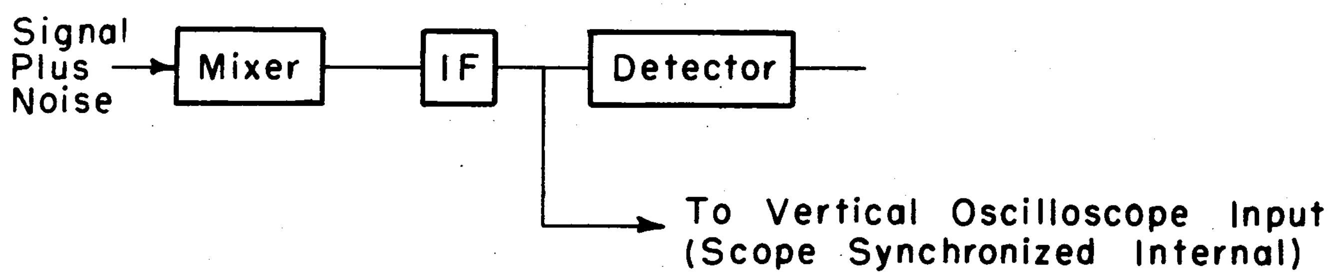


Fig. 2. Amplitude probability density function of
(a) Sine wave
(b) Narrow band Gaussian noise
(c) Broadband Gaussian noise.

IF strip such as would exist in a superhetrodyne receiver which was part of an overall phase-locked loop, as shown in Fig. 3. If the reference signal source were used to trigger the sweep of an oscilloscope, and the narrow-band noise existing in the IF strip were displayed, a presentation such as in Fig. 4(a) is obtained. When presented in this way, the intensity distribution in a cross section taken through the signal peak is given by Eq. (1), i.e., it is Rayleigh distributed. It is of interest to determine the point of peak intensity relative to the mean square

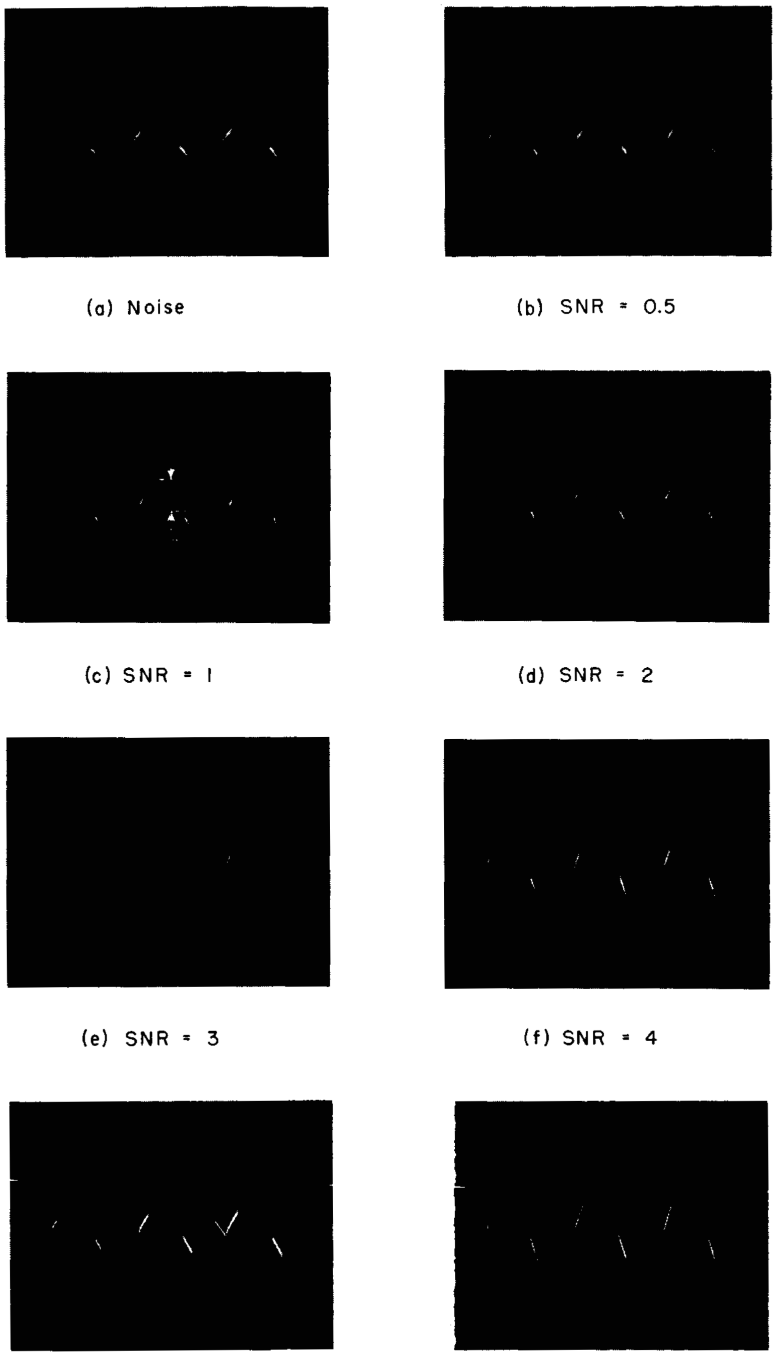


(a) External Synchronizing Operation



(b) Internal Synchronizing Operation

Fig. 3. Superheterodyne receiver in phase locked loop.



(a) Noise
(b) SNR = 0.5
(c) SNR = 1
(d) SNR = 2
(e) SNR = 3
(f) SNR = 4
(g) SNR = 5
(h) SNR = 20

(a) Thru (f) 0.05 Volt/cm
(g) Thru (h) 0.1 Volt/cm

Fig. 4. Oscilloscope display of noise and signal plus noise.

value of the waveform. This can be done by differentiating Eq. (1) with respect to the envelope amplitude. Thus

$$\frac{d\,q(r)}{dr} = \frac{e^{-r^2/2N}}{N} - \frac{r^2\ e^{-r^2/2N}}{N^2} \quad . \tag{2}$$

By equating this derivative to zero and solving for r, it is found that the peak intensity or population is obtained at $r = \sigma$, which is the rms voltage. Thus using a calibrated scale on the oscilloscope, the voltage deflection from the base line to the point of maximum intensity is equal to the rms value of the noise. The error involved in the measurements is due to the lack of a sharply defined point of maximum intensity; but by working with low overall intensity, a reasonably good estimate can be made of the rms value.

The next point of interest is centered on the envelope density function of a sinusoidal signal plus narrow-band noise. Again this distribution is well known and is given by[4]

$$q(r) = \frac{r}{N}\, e^{-(r^2 + A^2)/2N} \quad I_o\left(\frac{rA}{N}\right) \quad , \tag{3}$$

where r and N are as defined previously, A is the maximum value of the sine wave, and I_o is a modified Bessel function of the first kind[5]. By displaying the signal-plus-noise on an oscilloscope, as was done for the noise alone, a sinusoidal waveform is obtained which has noise imposed on it; the degree of the noise is dependent upon the SNR. Typical waveforms are shown in Figs. 4(b) - (h) for various SNR's. It is somewhat more difficult to obtain an analytical expression for the points of peak intensity as was done for the noise alone. This is due to the complexity of the derivative of the modified Bessel function. A somewhat simpler approach is to plot the distribution given by Eq. (3) for various SNR's. This is shown in Fig. 5 for SNR from 0.5 to 9. The peak of the curves, which correspond to points of peak intensity on the oscilloscope, do not coincide exactly with the signal-plus-noise voltage for the given SNR's; however the discrepancy is so slight that corrections to measured values would be smaller than the accuracy to which the results can be read initially. Therefore, for practical purposes, the point of peak intensity will be taken as the signal-plus-noise voltage. The SNR can then be calculated as the square of the ratio of signal-plus-noise voltage to rms noise voltage, and subtracting unity from the results. An interesting point for SNR's

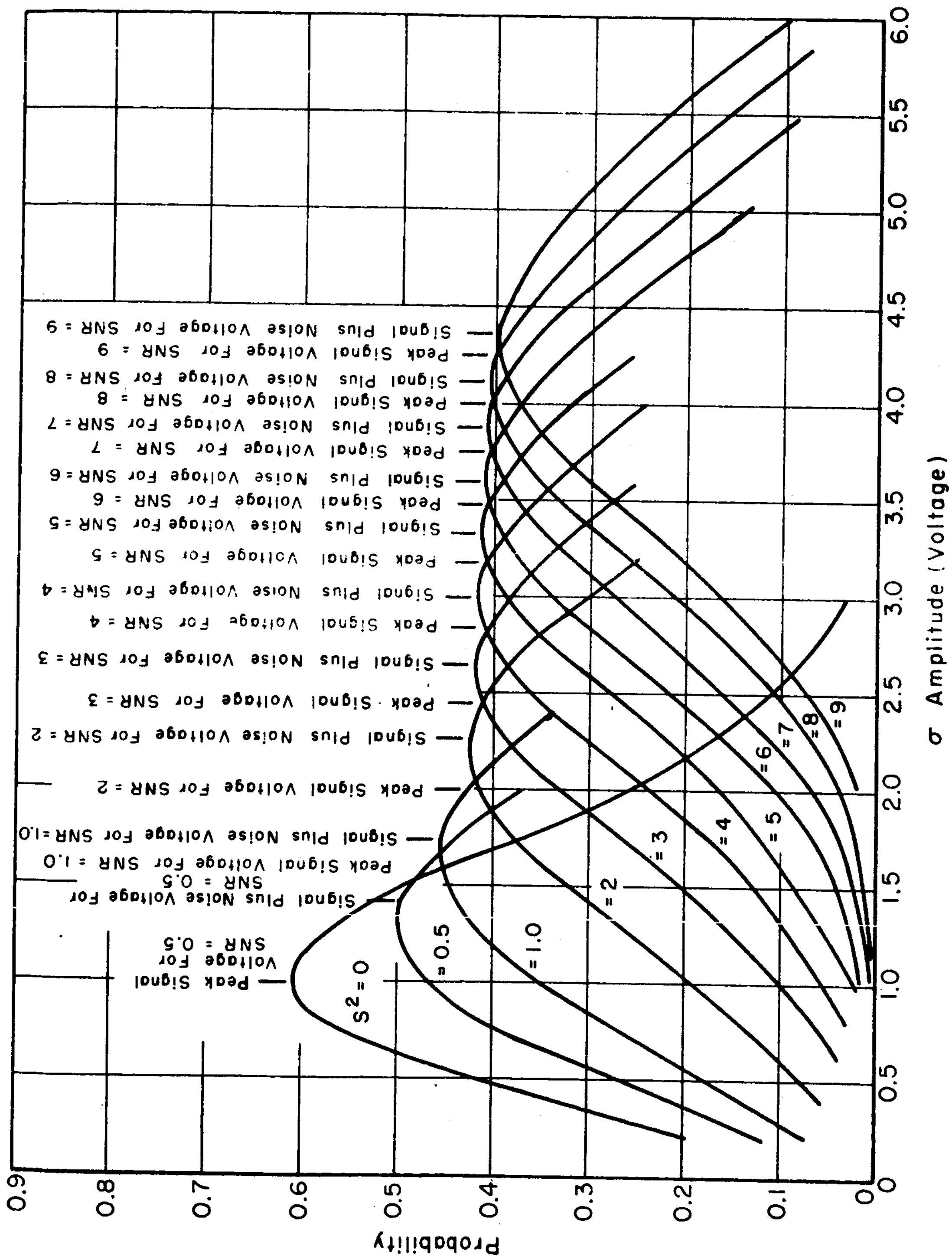


Fig. 5. Probability density vs rms amplitude of sinusoidal signal plus Gaussian noise for various signal to noise ratios.

greater than 5 is that the SNR can be read to an accuracy of better than 10% by taking the point of peak intensity to be the peak voltage. The reason for this is that as SNR increases, the curve becomes Gaussian distributed about the peak signal voltage, with the variance about this point then equaling the rms voltage[6]. In such case, central moments can be used to define the curve instead of the Rayleigh expression. The points of peak signal voltage are indicated in Fig. 5 along with the points of signal-plus-noise voltage, and it can be seen that they converge for large SNR's.

GENERAL COMMENTS

It should be noted that this measurement procedure will yield an SNR which is the ratio of mean peak signal power to mean noise power (mean signal power to mean noise power (SNR) is one half this ratio), with the signal power measured in the presence of noise and the noise power measured in the absence of signal. This is contrasted with the often-used criterion (when nonlinear demodulators are used) of measuring signal power with noise absent and noise power with signal absent. However, because of the measurement technique used here, the same result will obtain regardless of which definition is used because the measurement equipment is linear in its operation. As a result, the noise power (signal power) remains the same, regardless of whether it is measured in the presence or absence of signal (noise). This statement is made on the assumption that the mixers in the superheterodyne receiver are operated in their linear range with respect to the signal and noise voltages existing at the mixer input.

The measurement procedure described is particularly suited for the determination of carrier-to-noise ratios (CNR). When modulation is present on the signal, interpretation of the scope presentation becomes more difficult and as a result, accuracy is limited. However, the method proves useful as long as modulation percentage is not greater than 20% for AM.

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